

4/13/84  
1-11

**A TOOL FOR MANUAL ENDMEMBER SELECTION AND SPECTRAL UNMIXING**

C. Ann Bateson and Brian Curtiss

CIRES and Center for the Study of Earth from Space  
University of Colorado  
Campus Box 449  
Boulder, Colorado 80309-0449

**1.0. INTRODUCTION**

Sampling a continuous radiance spectrum in many narrow contiguous spectral bands results in a high covariance between the bands. Hence, the true dimensionality of imaging spectrometer data is not determined by the number of spectral bands, but by the number of spectrally unique signatures whose mixtures reproduce the spectral variance observed in an image. Methods to unmix high dimensional multispectral data use principal components analysis to reduce the dimensionality. The variance of the spectral data is modeled as a linear combination of a finite set of endmembers in the space of the eigenvectors that account for most of the variance. The number and characteristics of these endmembers are determined not only by the number and characteristics of the spectrally unique materials on the surface but also by processes (e.g., illumination, atmospheric scattering and absorption) affecting the signal received by the sensor.

Selection of endmember spectra has typically been from a library. However, since most libraries are incomplete and do not account for the processes mentioned above, we have devised a computer display that allows researchers to explore interactively the eigenvector space of a representative and mean-corrected subset of the image data in search of extreme spectra to designate as endmembers. This display, which is based on parallel coordinates (Inselberg, 85), is unique in the area of multidimensional visualization in that it includes not only a passive view of higher dimensional data but also the capability to interact and move geometrical objects in higher dimensional spaces.

**2.0. UNMIXING RECIPE**

After endmember selection, unmixing of the spectral data to compute concentrations is straight forward. If the true dimensionality of the data is n, then there are n+1 endmembers,  $E_1, \dots, E_{n+1}$  in the scene. The endmembers and spectral data are offset by  $E_1$  and rotated into the eigenvector space of dimension n. For each spectrum  $P = (p_1, \dots, p_n)$  in the data, solution of the matrix equation below yields the concentrations  $(c_1, \dots, c_{n+1})$ :

$$\begin{bmatrix} 0 & e_{12} & \dots & e_{1n} \\ \dots & \dots & \dots & \dots \\ 0 & e_{n2} & \dots & e_{nn} \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ \dots \\ c_n \\ c_{n+1} \end{bmatrix} = \begin{bmatrix} p_1 \\ \dots \\ p_n \\ 1 \end{bmatrix}$$

The vector  $(e_{1i}, \dots, e_{ni})$  is  $E_i$ , offset and rotated. Note that the last row of 1's guarantees that the concentrations sum to 1. A spectrum P viewed as a point in n dimensional space

PRECEDING PAGE BLANK NOT FILMED

lies within the  $n$ -simplex\* with  $E_1, \dots, E_{n+1}$  as vertices, if and only if each  $c_i$  lies between 0 and 1.

### 3.0. ENDMEMBER SELECTION IN LOWER DIMENSIONS

The endmember selection tool aids the researcher in finding endmembers as vertices of an  $n$ -simplex that contains all the spectral data. Figure 1 shows the display for the 3-dimensional case. Initially, the first panel of the four panel endmember selection screen shows the data projected into the space of the first two eigenvectors. As the cursor moves over this panel, the current spectrum window displays the spectrum of the point underneath. By clicking on the mouse, the user may select 3 endmembers in this window. If a fourth endmember is selected, then the plane of the first panel becomes three dimensional space and the fourth endmember is the apex of a tetrahedron whose base is the triangle in the plane. The four panels in figure 1 correspond to the four faces of the tetrahedron. The user may relocate endmembers in the panels in order to encompass the data within a simplex whose vertices are recognizable endmember.

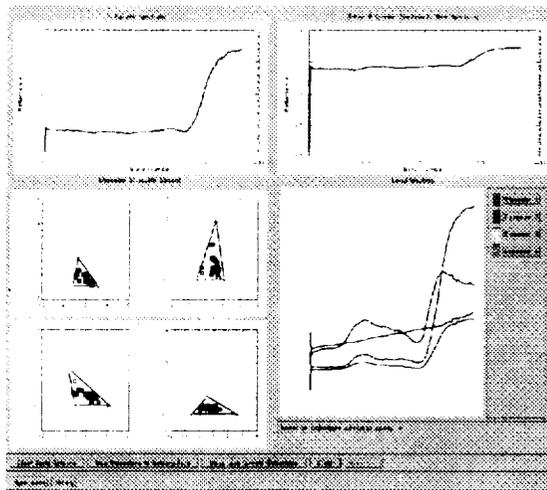


Figure 1. The endmember selection tool for three dimensions.

### 4.0. Higher Dimensional Endmember Selection

The endmember selection tool does not use the simple projection method above for dimensions exceeding 3 because of the following shortcomings:

a. The coordinate system becomes confusing when the data is projected. That is, it is hard to change an endmember spectrum only in the direction of a specified eigenvector

b. It is difficult to reach all points in a higher dimensional space by adjusting the vertices of the triangular faces of the constructed simplex.

c. The number of planes grows on the order of  $n^3$ , where  $n$  is the dimension of the space.

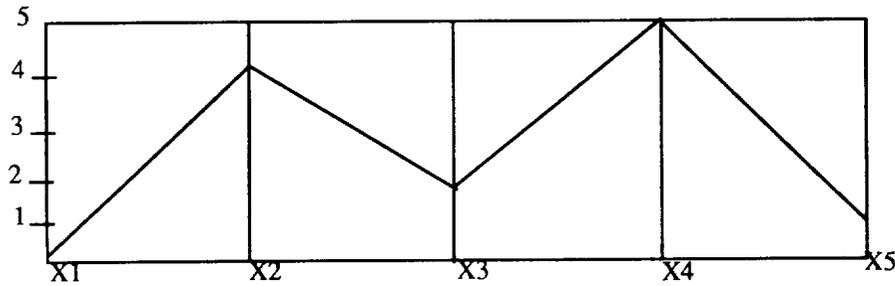
Parallel coordinate systems (Inselberg, 85) allowed the development of a display to overcome these shortcomings.

#### 4.1 BRIEF OVERVIEW OF PARALLEL COORDINATES

---

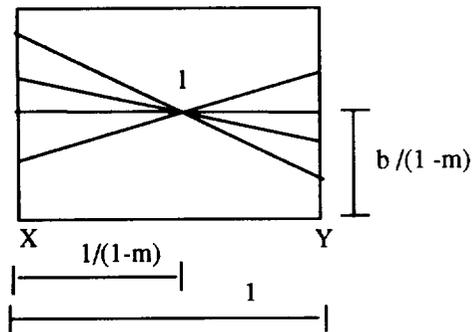
\*. That is, a triangle in the plane, a tetrahedron in 3-space and in  $n$ -space a polyhedron with  $n+1$  vertices and an edge for each vertex pair.

The parallel coordinate representation of the point (0, 4, 2, 5, 1) in fifth dimensional space is:



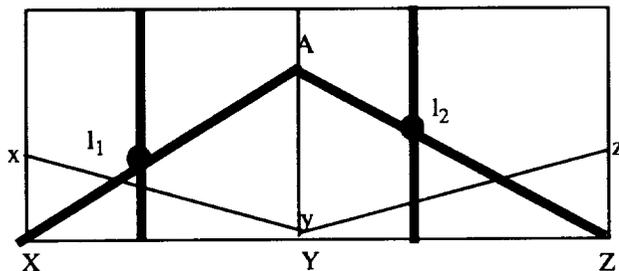
The five equally spaced vertical lines are the axes of the parallel coordinate system and the polygonal line segment is the plotted point. Note that spectra have a very natural representation in parallel coordinates with a parallel axis for each wavelength.

While points are represented by line segments in parallel coordinates, lines are dually represented as a collection of points. Below are points in parallel coordinates lying on the line  $y = m \cdot x + b$ :



The point  $l$  is said to represent the line  $y = m \cdot x + b$  in parallel coordinates. Observe that if  $l_1$  and  $l_2$  represent parallel lines then they lie on the same vertical line in parallel coordinate representation.

The parallel coordinate treatment of planes is based on the simple observation that a plane is determined by two lines  $l_1$  and  $l_2$  intersecting at a point  $A$ . The plot below shows the two points representing  $l_1$  and  $l_2$ , the polygonal line representing  $A$ , and a polygonal line representing another point on the plane. In this plot, we assume that  $A$  lies on the  $y$ -axis. In the higher dimensional display, the face of an  $n$ -simplex may be trans-



lated away from the vertex (endmember) opposite that face by adjusting  $A$ . The face may be rotated by adjusting the vertical lines through  $l_1$  or  $l_2$ .

## 4.2 TOOLS OF THE DISPLAY

The higher dimensional display includes the following tools:

1. The data window (figure 2) shows the selected endmembers and the parallel coordinate representation of the spectra projected into the eigenvector space. The user may adjust coordinate values of the endmembers in this window.

2. The cluster window (figure 2) has a parallel coordinate axis for each endmember. Concentrations are plotted on these axes. The user can look for and select clusters in this window. For example, the user could select the cluster containing all spectra on the wrong side of the hyperplane of the face opposite the first endmember--- i.e, all spectra which have a negative abundance for the first endmember.

3. The face window (figure 3) displays spectra selected in the cluster window. The medium width polygonal lines in the face window below are data on the wrong side of the face opposite a selected endmember. The thinner lines represent points on the hyperplane of the face. The single thickest polygonal line represents an endmember A and the thinner vertical lines are slope lines. Note that A and the thin polygonal lines intersect the second slope line at the same point. To relocate the hyperplane so that the spectra lie on the positive side, the user interactively adjusts A and the slope lines to reverse the relative location of the two sets of polygonal lines representing points on the hyperplane and those on the wrong side.

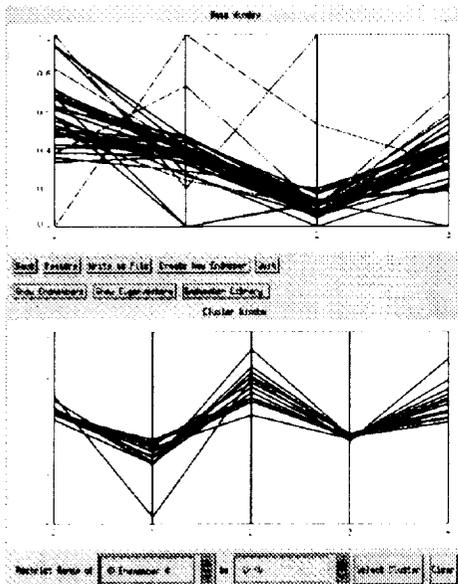


Figure 2. The data and cluster window

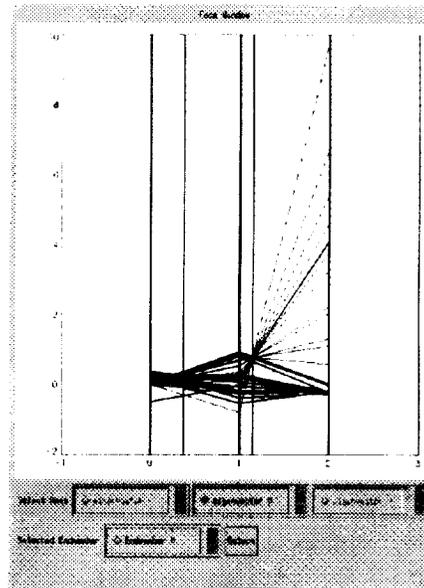


Figure 3. The face window

## REFERENCE

Adams, J.B., M.O. Smith and P.E. Johnson, 1986, "Spectral Mixture Modeling: A New Analysis of Rock and Soil Types at the Viking Lander Site, "J.Geophys. Res., 91, pop. 8098-8112.

Inselberg, A., 1985, "The Plane with Parallel Coordinates", Special Issue on Computational Geometry of the Visual Computer 1, pp.69-97.